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International Journal of Heat and Mass Transfer 47 (2004) 641–643
International Journal of HEAT and MASS TRANSFER
www.elsevier.com/locate/ijhmt

Letter to the Editor

Comments on "The onset of transient convection in bottom heated porous media", by K.-K. Tan, T. Sam and H. Jamaludin: Rayleigh and Biot numbers

Tan et al. [1] have written a significant paper on the onset of transient convection in a layer of a porous medium heated from below. However, their discussion is flawed at the outset (see the third paragraph on page 2859 of their paper) by their assumption that the Rayleigh number that arises in the stability analysis is that based on a thermal diffusivity κ_m defined as the effective thermal conductivity of the porous medium divided by the effective thermal capacity of the medium, rather than a thermal diffusivity κ^* (called the "modified thermal diffusivity" in [1]) defined as the effective thermal conductivity of the medium divided by the thermal capacity of the fluid. That the latter is in fact the correct quantity can be seen immediately from the ratio of coefficients in the steady state form of the thermal energy equation (see, for example Eq. (6.5) of [2])

$$(\rho c_P)_f \mathbf{v} \cdot \nabla T = k_m \nabla^2 T. \qquad (1)$$

Here T is the temperature, $\mathbf{v}$ is the Darcy velocity, k_m is the effective thermal conductivity of the medium, while $(\rho c_P)_f$ is the thermal capacity of the fluid. The important point is that the heat is convected in the fluid phase only, whereas it is conducted in both the fluid and solid phases.

Thus Elder [3] had no reason to "mention the reason for employing the modified diffusivity"—it was just the sensible thing to do. Also, it is clear that he has not "mistaken the thermal boundary to be of the FST [fixed surface temperature] boundary condition". In fact, since he obtained an experimental value for the critical Rayleigh number of about 40 (compared with the theoretical value $4\pi^2$ that applies for the case of FST and impermeable boundaries) it is clear that in his experiment the FST condition must have been approximated quite well. Similarly, the fact that Chen and Chen [4] obtained an experimental value of 40.07 for the critical Rayleigh number shows that the FST condition, and not the CHF condition, applied in their experiments.

Likewise it is not correct that Katto and Masuoka [5] suggested the use of κ^* for calculating the Rayleigh number just because their experimental results for nitrogen gas agreed with the theoretical critical Rayleigh number of 39.4. They had a sound theoretical reason for this usage.

Incidentally, the authors of [1] are mistaken in their belief that "Ribando and Torrance [6] were the first researchers to extend Lapwood's analysis to the case of bottom heating with a CHF (constant heat flux) boundary condition and provided the theoretical values ... as shown in Table 1" (of [1]). The values were obtained some eight years earlier by Nield [7]. (Actually Nield [7] reported the values 27.10 and 17.65 rather than 27.1 and 17.7.)

More importantly, the authors of [1] stated that "There are no known theoretical studies of the onset of convection in porous media caused by unsteady-state heat conduction". They have overlooked the paper by Nield [8], which contains a section on convection in porous media. Nield obtained estimates of values of the critical Rayleigh number for CHF boundaries for the cases of (a) two impermeable boundaries, and (b) one boundary impermeable and the other permeable.

Tan et al. further stated that they had estimated that "the Biot number in most of the bottom heating experiments using a plate heater could only be characterized by a CHF boundary condition as the fluid is rather insulating relative to the heater and typically a system of glass–water matrix heated by a copper heater will yield a Biot number of approximately 0.09". This is very surprising, because one would then expect a large rather than a small value of the Biot number. Tan et al. [1] do not give the expression from which they calculated the Biot number. In a like manner, Tan et al. [1] criticized the study by Shattuck et al. [9], claiming that "They erroneously used $Ra_c = 4\pi^2$" and that the Biot number "may be easily determined to be about 0.05 as the highly conducting ceramic heater ... has a high conductivity". From a private communication from Dr. Tan the author has learnt that Tan et al. [1] followed Pearson [10] and used a Biot number defined by the heat transfer ratio $(\mathrm{d}q/\mathrm{d}T)_{\mathrm{top}}/(\mathrm{d}q/\mathrm{d}T)_{\mathrm{bottom}}$ at the interface. They should have used the ratio $(\mathrm{d}q/\mathrm{d}T)_{\mathrm{exterior}}/(\mathrm{d}q/\mathrm{d}T)_{\mathrm{interior}}$. For the Pearson problem involving the Marangoni effect the relevant interface is the upper surface, and the two ratios are equivalent. However, for the problem considered in [1] the relevant interface is the lower surface, and the two ratios are the reciprocals of each other. In contrast to the Nusselt number, the Biot number is a measure of relative conductivity (or relative resistivity). It is worth

0017-9310/$ - see front matter
doi:10.1016/j.ijheatmasstransfer.2003.07.016

noting that Biot worked on conduction some decades before Nusselt worked on convection.

Also in connection with [9], Tan et al. [1] state that "the bulk fluid of the porous media (sic) exerts a substantial shear so that a free surface is not attainable in the presence of the solids". In the context of a porous medium governed by Darcy's law this statement does not make sense, because the Darcy equation is consistent with slip on the boundaries. Tan et al. [1] also stated that in the experiments reported in [9] were for a particle diameter to depth ratio too small for the Darcian flow to be homogeneous. This is irrelevant. It is not "the main cause of the deviations of experiments from theory".

Tan et al. [1] also refer to the anomalous result of Kaneko et al. [11], who obtained a critical Rayleigh number of 28, which Tan et al. noted was close to the theoretical value of 27.1 for the CHF boundary. It is likely that this is just a coincidence, and that the discrepancy is simply due to a nonlinear basic temperature profile (something pointed out in [2]).

Further, Tan et al. state "It is clear that the Rayleigh number can only be calculated at a very strict condition where the thermal diffusivity of the solid and liquid matrix are similar and the permeability of the porous matrix should be large enough so that ΔT_s or heat flux will be small". This statement is incorrect.

In summary, Tan et al. [1] have misinterpreted virtually all the previous experimental work on this topic. Their own work thus requires reinterpretation. Fortunately they have provided some alternative values of the critical Rayleigh number and Nusselt number (e.g. in the last column of each of their Tables 4–7) that are valid. However, it appears that their predicted critical time values are invalid.

I am grateful for a private communication from Dr. Tan.

Addendum on the Biot number

The term 'Biot number' or 'Biot modulus' has been in use for several decades in the context of films on solid slabs. The term is named after Jean Baptiste Biot (1774–1862), after whom the Biot–Savart law in electromagnetism and the Biot law in optics are also named. The appellation of 'Biot number' to the parameter that appears as a boundary condition in the stability problem for Rayleigh–Bénard convection is apparently due to Sparrow et al. [12]. In the same year (1964) of that publication, Scriven and Sternling [13] called the parameter a Nusselt number, while earlier (1958) Pearson [10] had simply called the parameter L. Professor Antony Pearson (private communication) has confirmed that he was unaware of the previous usage of the parameter at the time he wrote his paper. Professor Richard Goldstein (private communication) has explained the somewhat novel nomenclature used in [12] as follows (see the next three paragraphs).

Both the Nusselt and Biot numbers contain the three parameters h, l and k. With the Nusselt number they refer to the fluid undergoing motion with convection occurring. The Nusselt number is then the ratio of the convective heat transport to the conduction heat transfer that would occur through a stagnant layer of fluid of thickness l. The h, l, k in the Biot number usually refer to the actual heat transfer coefficient (as for Nu,) but k and l refer to the conductivity and thickness of the bounding solid. Thus the Biot number is the ratio of the thermal resistance across the solid boundary (assuming beyond the thickness, l, there is a uniform temperature) to that across the fluid boundary layer.

The Biot number has typically been used for conduction analysis where all three parameters (and then, of course, the Biot number itself) are given independent variables. In contrast the Nusselt number (and h) is almost always a dependent variable.

In the stability case we first consider the fluid as a conducting solid (really just no flow) and the parameters h, l, k are essentially used as in a transient conductor analysis. All are known independent variables. The tricky part may be that h is not a true convective coefficient, but rather a measure of the inverse of the thermal resistance at the solid boundary. The parameters l and k refer to the potentially convecting fluid layer. Thus hl/k is a Biot number relating the thermal boundary condition on the fluid/solid interface, varying from constant temperature (Bi tends to infinity) to constant heat flux (Bi tends to zero).

Equation (1) is based on averaging over a representative elementary volume containing the fluid and solid phases. The classic paper by Lapwood is misleading (if not erroneous) because he did not explicitly define the thermal diffusivity.

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